Radical Functions and Transformations

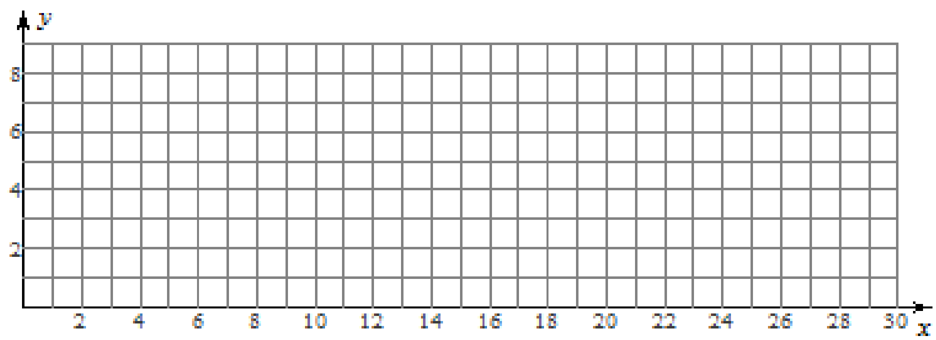
A radical function is one in which the variable occurs in the radicand. Radical functions have restricted domains if the index of the radical is an even number. The domain of $y=\sqrt{f(x)}$ consists only of the values in the domain of $f(x)$ for which $f(x)\geq 0$. The range of $y=\sqrt{f(x)}$ consists of the square roots of the values in the range of $y=f(x)$ for which $y=\sqrt{f(x)}$ is defined. Examples of radical functions include $y=\sqrt{3-x}$, $y=2\sqrt{3x+1}-5$, and $y=\sqrt{x^2-25}$.

Example 1: Graph Radical Functions Using Tables of Values

Use a table of values to sketch the graph of $y=\sqrt{x}$. Then, state the domain, range, and intercepts of the function.

Solution:

$y=\sqrt{x}$	
x	y
0	
1	
4	
9	
16	
25	



Domain: ______________________________

Range: ______________________________

x-intercept: _____________

y-intercept: _____________

Just like other functions, radical functions can be transformed. The shape of transformed radical functions is very similar to the basic radical function, however, the placement and orientation may be different. The effects of changing the parameters in radical functions are the same as the effects of changing parameters in other types of functions:

$y=\sqrt{x}$ can be transformed to $y=a\sqrt{b(x-h)}+k$

Example 2: Graph Radical Functions Using Transformations

Sketch the graph of each function using transformations. Determine the domain and range of each function.

a. $y = 3\sqrt{-(x+1)}$

b. $y = \sqrt{2x} - 3$

c. $f(x) = -2\sqrt{4(x-2)} - 5$

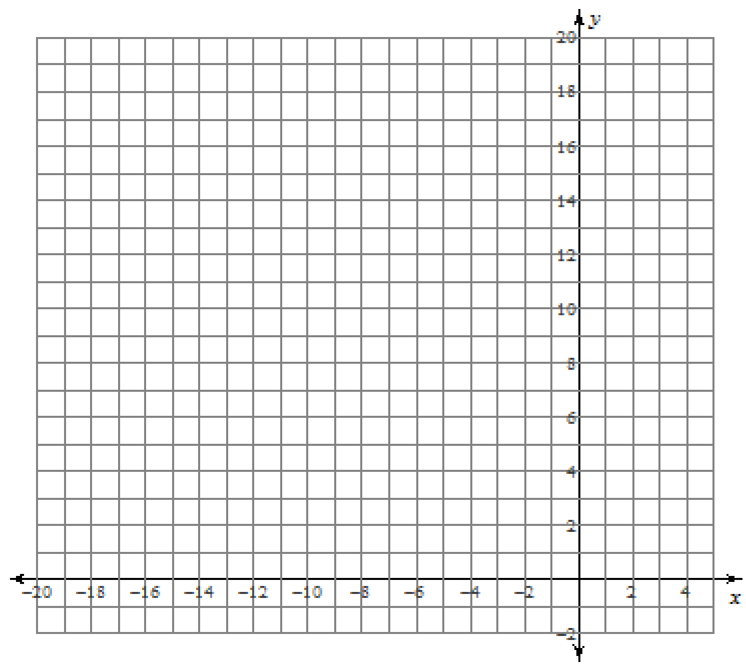
Solution:

a. $y = 3\sqrt{-(x+1)}$

$y = \sqrt{x}$	
x	y
0	
1	
4	
9	
16	

Mapping Rule: ____

$y = 3\sqrt{-(x+1)}$	
x	y



Domain of $y = 3\sqrt{-(x+1)}$: ______________________________

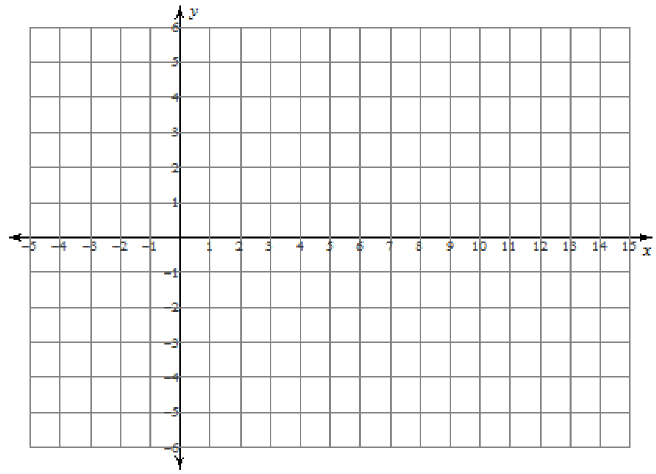
Range of $y = 3\sqrt{-(x+1)}$: ______________________________

b. $y = \sqrt{2x} - 3$

Mapping Rule: ______________________________

$y = \sqrt{x}$	
x	y
0	0
1	1
4	2
9	3
16	4
25	5

$y = \sqrt{2x} - 3$	
x	y



Domain of $y = \sqrt{2x} - 3$: ______________________________

Range of $y = \sqrt{2x} - 3$: ______________________________

c. $f(x) = -2\sqrt{4(x-2)} - 5$

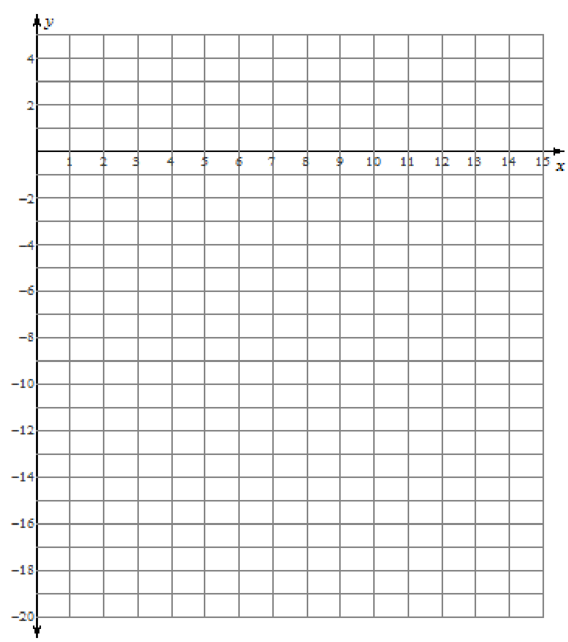
Mapping Rule: ______________________________

Domain of $f(x) = -2\sqrt{4(x-2)} - 5$: ______________________________

Range of $f(x) = -2\sqrt{4(x-2)} - 5$: ______________________________

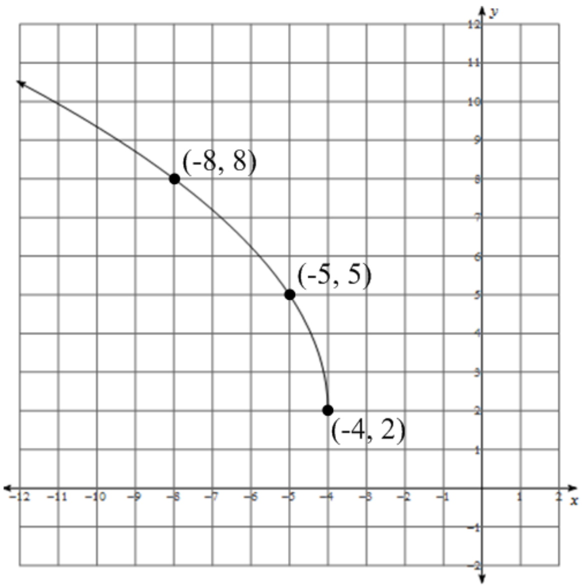
$y = \sqrt{x}$	
x	y
0	0
1	1
4	2
9	3
16	4

$f(x) = -2\sqrt{4(x-2)} - 5$	
x	y



Example 3: Determine a Radical Function From a Graph

Use the graph below to write the equation of the corresponding radical function in the form $y=a\sqrt{b(x-h)}+k$



Solution:

You can use the endpoint and the coordinates of another point of the transformed function to determine the equation.

Identify the endpoint of the transformed function: __________

The endpoint represents the parameters _____ and _____.

Reflection in the x-axis: __________

Reflection in the y-axis: __________

Equation (so far): __________________________

The base graph $y=\sqrt{x}$ has been stretched. This stretch can be viewed as either a horizontal or vertical stretch. To determine the factor by which the graph has been stretched, begin by identifying one other point that the graph passes through.

View as a Vertical Stretch	View as a Horizontal Stretch
Another point: _________	Another point: _________
Substitute the coordinates of this point into $y=a\sqrt{-(x+4)}+2$ and solve for a.	Substitute the coordinates of this point into $y=\sqrt{-b(x+4)}+2$ and solve for b.
Equation of the function: ____	**Equation of the function:** ____